Problem Set 2

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2.1 During lecture 1, we discussed the simple energy balance model

$$C\frac{dT_s}{dt} = (1-\alpha)Q - \varepsilon\sigma T_s^4$$

where C is the specific heat of the system, T_s is the global mean surface temperature, α is the global albedo, Q is the annual mean global mean solar flux, ε represents the effect of atmospheric greenhouse gases and $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan–Boltzmann constant.

(a) Construct a function that calculates $\frac{dT_s}{dt}$. Include C, α, Q and ε as parameters with the default values $C = 2.109 \times 10^8\,\mathrm{J\,K^{-1}\,m^{-2}}$, $\alpha = 0.3$, $Q = 342\,\mathrm{W\,m^{-2}}$ and $\varepsilon = 0.615$.

(b) Suppose ε decreases at a rate of 0.1% per year from an initial value of $\varepsilon_0 = 0.615$. Plot the evolution of surface temperature over 100 years, assuming that $T_s = 288\,\mathrm{K}$ at $t = 0$. Be careful about units, and beware of large time steps!

(c) Use periodic functions with periods of 26 000, 41 000, 100 000 and 400 000 years to modify the solar parameter Q. The amplitudes of the functions are up to you (see what happens when you vary them), but aim for a minimum Q of about 80% of present-day and a maximum Q of about 130% of present-day. Use these simulated variations in Q to calculate and plot the evolution of temperature over one million years.

(d) Modify your model so that the albedo α is a function of temperature: $\alpha(T) = 0.5 - 0.2\tanh\left(\frac{T-265}{10}\right)$. Recalculate the time series of T_s from part (c) using this modified model. How are the results different? Why?

(e) **Extra credit:** revisit part (b). Estimate how many years it takes for the system to reach equilibrium if ε stops increasing after year 100. What is the new equilibrium temperature?

(f) **Extra credit:** this website allows you to view estimated changes in the solar constant over millions of years. How do these changes compare to the changes you simulated in part (c)? Download some subset of these data, read them using the `csv` module, use them as input to your energy balance model and plot the results. Do you think that these results give a more or less realistic picture of actual climate variations than your results from parts (c) and (d)? Explain.

2.2 Now construct a version of the two-box energy model:

$$\mu_\mathrm{m}\frac{dT_\mathrm{m}}{dt} = (1-\alpha)Q - \kappa(T_\mathrm{m} - T_\mathrm{o}) - \varepsilon\sigma T^4$$
$$\mu_\mathrm{o}\frac{dT_\mathrm{o}}{dt} = \kappa(T_\mathrm{m} - T_\mathrm{o})$$

with default values $\alpha = 0.3$, $Q = 342\,\mathrm{W\,m^{-2}}$, $\kappa = 0.7\,\mathrm{W\,m^{-2}\,K^{-1}}$, $\rho_\mathrm{m} = 1025\,\mathrm{kg\,m^{-3}}$, $\rho_\mathrm{o} = 1035\,\mathrm{kg\,m^{-3}}$, $c_p = 3850\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $\varepsilon = 0.615$ and σ taken from `scipy.constants`. Use any of the methods for solving dynamical systems of equations described in the notes.

(a) Calculate the initial equilibrium temperature T_0 based on the default parameter values. You should use T_0 as the initial condition for your later simulations.

(b) Suppose that a massive volcanic eruption occurred, suddenly increasing the global mean albedo α to 0.4 and reducing the effective transmissivity ε to 0.58. If these conditions persisted, what would the new equilibrium temperature be? Calculate and plot the evolution of T_m and T_o in time until surface temperature is within 0.1 K of this new equilibrium. Assume that both T_m and T_o were in equilibrium before the sudden change occurred. Decide on appropriate values for the depths of the two ocean layers (D_m and D_o), but use default values for all other parameters.

(c) Changing only parameter values in your function call (i.e., without changing the function itself), redo part (a) for the one-box case with depth D_m. Check your solution by adding a plot of the linearized approximation $T'(t) = T'_0 \exp\left(-t/\tau\right)$ to the same axes.

(d) **Extra credit:** rewrite your code to allow α and ε to decay exponentially back toward their default values at different timescales (e.g., $\tau = 5$ years for α and $\tau = 100$ years for ε). Plot the evolution of temperature for the same values of D_m and D_o that you used in part (b).

(e) **Extra credit:** compare your results to a two-box model with $OLR = A + B(T_\mathrm{m} - 273.15)$, with $A = 203.3\,\mathrm{W\,m^{-2}}$ and $B = 2.09\,\mathrm{W\,m^{-2}\,{}^\circ C^{-1}}$ (the Budyko linearization mentioned in lecture 1).

2.3 The unified oscillator is a model of El Niño–Southern Oscillation variability that combines four idealized models of physical feedbacks in the tropical Pacific Ocean. This model is expressed as a system of four differential equations:

$$\frac{\partial T}{\partial t} = a\tau_1(t) - b_1\tau_1(t-\eta) + b_2\tau_2(t-\delta) - b_3\tau_1(t-\mu) - \varepsilon T(t)^3$$
$$\frac{\partial h}{\partial t} = -c\tau_1(t-\lambda) - R_h h(t)$$
$$\frac{\partial \tau_1}{\partial t} = dT(t) - R_{\tau_1}\tau_1(t)$$
$$\frac{\partial \tau_2}{\partial t} = eh(t) - R_{\tau_2}\tau_2(t)$$

Here T is the Niño3 sea surface temperature anomaly, h is the Niño6 thermocline depth anomaly (here represented by the average temperature between the surface and 300 m depth), τ_1 is the zonal wind stress anomaly in the Niño4 region, and τ_2 is the zonal wind stress anomaly in the Niño5 region. The parameters $\eta = 150\,\mathrm{d}$, $\delta = 30\,\mathrm{d}$, $\mu = 90\,\mathrm{d}$, and $\lambda = 180\,\mathrm{d}$ are delay terms that account for time lags in the physical system. Wang (2001) suggested that the other parameters could be specified as:

$a = 0.41\,\mathrm{K\,m^2\,N^{-1}\,d^{-1}}$	$b_1 = 0.68\,\mathrm{K\,m^2\,N^{-1}\,d^{-1}}$
$b_2 = 2.1\,\mathrm{K\,m^2\,N^{-1}\,d^{-1}}$	$b_3 = 0.68\,\mathrm{K\,m^2\,N^{-1}\,d^{-1}}$
$c = 0.91\,\mathrm{m^2\,N^{-1}\,d^{-1}}$	$d = 1 \times 10^{-4}\,\mathrm{N\,m^{-2}\,K^{-1}\,d^{-1}}$
$e = 3.3 \times 10^{-5}\,\mathrm{N\,m^{-2}\,K^{-1}\,d^{-1}}$	$\varepsilon = 3.3 \times 10^{-3}\,\mathrm{K^{-2}\,d^{-1}}$
$R_h = 3.1 \times 10^{-3}\,\mathrm{d^{-1}}$	$R_{\tau_1} = R_{\tau_2} = 5.5 \times 10^{-3}\,\mathrm{d^{-1}}$

(a) Write a program that uses forward Euler on a 1-day time step to integrate the unified oscillator. Use the provided data as the initial conditions (note that at least 181 days of data are required as initial conditions due to the delay terms), integrate the model for at least 20 years, and plot the results for the sea surface temperature anomaly (T).

(b) Identify the dominant period of the oscillator, and then look up the observed period of ENSO variability. How do these two periods compare? Identify and describe at least two differences between the unified oscillator model and the real ENSO.

(c) **Extra credit:** Solve part (a) using one of the integration methods in scipy.integrate.